SHIFTING PROPERTIES OF FINITE COSINE HYPERBOLIC TRANSFORMS

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ABSTRACT

A RAM Finite Hyperbolic Transforms are defined in [1]. Moreover these transforms of some standard functions are obtained in the same paper. In this paper we proved some shifting properties of RAM finite cosine hyperbolic transform.

Keywords: *Generalized Transform, Finite transform, Finite hyperbolic transform, properties of finite transforms.*

1. INTRODUCTION:

If the disturbance is f(t)= e^{at^2}, for a >0, the usual Laplace transform cannot be used to find the solution of an initial value problem because Laplace transform of f(t) does not exist. It is often true that the solution at times later than t would not affect the state at time t. This leads to define Finite Laplace transform.

The finite Laplace transform of a continuous or an almost piecewise continuous function f in (0,T) is denoted by $L_T(f(t)) = F(p,T)$, and is defined as

$$L_T(f(t)) = F(p,T) = \int_0^T f(t)e^{-pt}dt \tag{1.1}$$

where p is a real or complex number and T be a finite number which may be positive or negative. RAM finite sine and cosine hyperbolic transforms are defined in [1] as follows:

Definition 1.1 [1]: Let p$\in$ C and T be a finite number which may be positive or negative and f is a continuous or an almost piecewise continuous function defined over the interval (0,T). Then RAM Finite Sine Hyperbolic transform of f is denoted by Rsh (f(t)) = FS(p,T), and is defined as

$$R_{sh}(f(t)) = F_S(p,T) = \int_0^T \sinh(pt)\, f(t)dt$$

where sinh(pt) is a Kernel of Rsh.

Here Rsh is called RAM Finite Sine Hyperbolic transformation operator.

Definition 1.2 [1]: Let $p \in$ C and T be a finite number which may be positive or negative and f is a continuous or an almost piecewise continuous function defined over the interval (0,T). Then

RAM Finite Cosine Hyperbolic transform of f is denoted by
$R_{ch}(f(t)) = F_C(p,T)$, and defined by

$$R_{ch}(f(t)) = F_C(p,T) = \int_0^T \cosh(pt) f(t)\,dt$$

where $\cosh(pt)$ is a Kernel of R_{ch}.
Here R_{ch} is called RAM Finite Cosine Hyperbolic transformation operator.
In this paper we obtained some properties of finite hyperbolic cosine transform. Shifting properties are obtained in section [3], examples on these properties are obtained in section [4].

2. PRELIMINARIES:

2.1 RAM Finite Sine Hyperbolic Transform of some standard functions [1]

1 $R_{sh}(1) = \dfrac{\cosh(pT)-1}{p}$

2 $R_{sh}(t) = \dfrac{T\cosh(pT)}{p} - \dfrac{\sinh(pT)}{p^2}$

3 $R_{sh}(t^2) = \dfrac{T^2\cosh(pT)}{p} - \dfrac{2T\sinh(pT)}{p^2} + \dfrac{(2\cosh(pT)-2)}{p^3}$

4 $R_{sh}(t^k) = \begin{cases} \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{p^2} + ... + \dfrac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if } k \text{ is even} \\ \dfrac{T^k\cosh(pT)}{p} - \dfrac{kT^{k-1}\sinh(pT)}{p^2} + ... + \dfrac{k!(-1)^k\sinh(pT)}{p^k}, \text{if } k \text{ is odd} \end{cases}$

5 $R_{sh}(\sin(at)) = \left(\dfrac{-a}{p^2+a^2}\right)\sinh(pT)\cos(aT) + \left(\dfrac{p}{p^2+a^2}\right)\cosh(pT)\sin(aT)$.

6 $R_{sh}(\cos(at)) = \left(\dfrac{a}{p^2+a^2}\right)\sinh(pT).\sin(aT) + \left(\dfrac{p}{p^2+a^2}\right)[\cosh(pT).\cos(aT) - 1]$.

7 $R_{sh}(e^{at}) = \left(\dfrac{-a}{p^2-a^2}\right)\sinh(pT).e^{aT} + \left(\dfrac{p}{p^2-a^2}\right)[\cosh(pT).e^{aT} - 1]$, provided $p^2 \neq a^2$.

8 $R_{sh}(e^{-at}) = \left(\dfrac{a}{p^2-a^2}\right)\sinh(pT).e^{-aT} + \left(\dfrac{-p}{p^2-a^2}\right)[1-\cosh(pT).e^{-aT}]$, provided $p^2 \neq a^2$.

2.2 RAM Finite Cosine Hyperbolic Transform of some standard functions [1]

1 $R_{ch}(1) = \dfrac{\sinh(pT)}{p}$.

2 $R_{ch}(t) = \frac{T\sinh(pT)}{p} - \left(\frac{\cosh(pT)-1}{p^2}\right)$

3 $R_{ch}(t^2) = \frac{T^2.\sinh(pT)}{p} - \frac{2.T.\cosh(pT)}{P^2} + \frac{2.\sinh(pT)}{p^3}$.

4 $R_{ch}(t^k) = \begin{cases} \frac{T^k\sinh(pT)}{p} - \frac{kT^{k-1}\cosh(pT)}{p^2} + ... + \frac{k!(-1)^k\sinh(pT)}{p^k}, \text{if k is even,} \\ \frac{T^k\sinh(pT)}{p} - \frac{kT^{k-1}\cosh(pT)}{p^2} + ... + \frac{k!(-1)^k[\cosh(pT)-1]}{p^k}, \text{if k is odd.} \end{cases}$

5 $R_{ch}(\sin(at)) = \left(\frac{a}{p^2+a^2}\right)[1 - \cosh(pT)\cos(aT)] + \left(\frac{p}{p^2+a^2}\right)\sinh(pT)\sin(aT)$.

6 $R_{ch}(\cos(at)) = \left(\frac{a}{p^2+a^2}\right)\cosh(pT)\sin(aT) + \left(\frac{p}{p^2+a^2}\right)\sinh(pT)\cos(aT)$.

7 $R_{ch}(e^{at}) = \left(\frac{a}{p^2-a^2}\right)[\cosh(pT)\, e^{aT} - 1] + \left(\frac{p}{p^2-a^2}\right)\sinh(pT)\, e^{aT}$, provided $p^2 \neq a^2$.

8 $R_{ch}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right)\cosh(pT)\, e^{-aT} + \left(\frac{-p}{p^2-a^2}\right)[1 - \sinh(pT)\, e^{-aT}]$, provided $p^2 \neq a^2$.

2.3 Some Properties of RAM Finite Cosine Hyperbolic Transform [1]

1 Linearity: $R_{ch}(f_1(t) + f_2(t)) = R_{ch}(f_1(t)) + R_{ch}(f_2(t))$

2 Scalar Multiplication: If c is any constant, then $R_{ch}(c.\, f(t)) = c.\, R_{ch}(f(t))$

3 Scaling: If $R_{ch}(f(t)) = F_C(p, T)$, then $R_{ch}(f(at)) = \frac{F_C\left(\frac{p}{a}, aT\right)}{a}$

3. SHIFTING PROPERTIES OF RAM FINITE COSINE HYPERBOLIC TRANSFORM:

Theorem 3.1: If $R_{sh}(f(t)) = F_S(p,T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{ch}(\cosh(at).\, f(t)) - R_{sh}(\sinh(at).\, f(t)) = F_C((p-a), T)$$

Proof: Let $R_{ch}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p,T)$, then

$$R_{ch}(\cosh(at).\, f(t)) - R_{sh}(\sinh(at).\, f(t)) = \int_0^T [f(t)(\cosh(at).\cosh(pt) - \sinh(at).\sinh(pt))]dt$$

$$= \int_0^T f(t)\cosh((p-a)t)\, dt$$

$$= F_C\,((p - a), T)$$

Theorem 3.2: If $R_{sh}\,(f(t)) = F_S(p,T)$ and $R_{ch}\,(f(t)) = F_C(p, T)$, then

$$R_{sh}\,(\sinh(at).\,f(t)) + R_{ch}\,(\cosh(at).\,f(t)) = F_C\,((p + a), T)$$

Proof: Let $R_{sh}\,(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{sh}(\sinh(at).\,f(t)) + R_{ch}(\cosh(at).f(t)) = \int_0^T [\,f(t)\,(\sinh(pt).\,\sinh(at) + \cosh(at).\,\cosh(pt))]\,dt$$

$$= \int_0^T f(t)\,(\cosh(p + a)t)\,dt$$

$$= F_C\,((p + a), T)$$

Theorem 3.3: If $R_{sh}\,(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p,T)$, then

$$R_{ch}(\cosh(at).f(t)) = \frac{F_C((p+a),T + F_C((p-a),T)}{2}$$

Proof: Using Theorem 3.1 and Theorem 3.2, we have,

$$R_{sh}(\sinh(at).f(t)) + R_{ch}\,(\cosh(at).f(t)) = F_C((p + a),T) \text{ and}$$

$$R_{ch}(\cosh(at).f(t)) - R_{sh}\,(\sinh(at).f(t)) = F_C((p - a),T)$$

$$\Rightarrow R_{ch}\,(\cosh(at).f(t)) = \frac{F_C((p-a),T) + F_C((p+a),T)}{2}$$

Theorem 3.4 : If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{ch}\,(\sinh(at).\,f(t)) = \frac{-F_S((p-a),T) + F_S((p+a),T)}{2}$$

Proof: Using Theorem 3.1 and Theorem 3.2, we have,

$$R_{ch}\,(\sinh(at).f(t)) - R_{sh}\,(\cosh(\underline{at}).\,f(t)) = -F_S\,((p - a), T) \text{ and}$$

$$R_{sh}\,(\cosh(at).f(t)) + R_{ch}\,(\sinh(at).\,f(t)) = F_S\,((p + a), T)$$

$$\Rightarrow R_{ch}\,(\sinh\,(at).f(t)) = \frac{-F_S((p-a),T) + F_S((p+a),T)}{2}$$

Theorem 3.5: If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}\,(f(t)) = F_C(p, T)$, then

$$R_{ch}\,(e^{-at}.\,f(t)) = \frac{F_C((p-a),T) + F_C((p+a),T) + F_S((p-a),T) - F_S((p+a),T)}{2}$$

Proof: Using Theorem 3.3 and Theorem 3.4, we have,

$$R_{ch}\,(\cosh(at).\,f(t)) = \frac{F_C((p-a),T) + F_C((p+a),T)}{2} \text{ and}$$

$$R_{ch}\,(\sinh(at).f(t)) = \frac{-F_S((p-a),T) + F_S((p+a),T)}{2}$$

$$\Rightarrow R_{ch}(e^{-at}\,.\,f(t)) = \frac{F_C((p-a),T) + F_C((p+a),T) + F_S((p-a),T) - F_S((p+a),T)}{2}$$

Theorem 3.6: If $R_{sh}\,(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{ch}(\cos(at).\,f(t)) = \frac{F_C((p-a),T)+F_C((p+a),T)-F_S((p-a),T)-F_S((p+a),T)}{2}$$

Proof: Using Theorem 3.3 and Theorem 3.4, we have,

$$R_{ch}(\cosh(at).\,f(t)) = \frac{F_C((p-a),T)+F_C((p-a),T)}{2} \text{ and}$$

$$R_{ch}(\sinh(at).f(t)) = \frac{-F_S((p-a),T)+F_S((p+a),T)}{2}$$

$$\Rightarrow R_{ch}(\cos(at).\,f(t)) = \frac{F_C((p-a),T)+F_C((p+a),T)-F_S((p-a),T)-F_S((p+a),T)}{2}$$

Theorem 3.7: If $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p,T)$, then

$$R_{ch}(\cos(at).f(t)) = \frac{F_C((p+ia),T)+F_C((p-ia),T)}{2}$$

Proof: Using Theorem 3.3, we have

$$R_{ch}(\cosh(at).f(t)) = \frac{F_C((p+a),T)+F_C((p-a),T)}{2}$$

$$\Rightarrow R_{ch}(\cosh(iat).f(t)) = \frac{F_C((p+ia),T)+F_C((p-ia),T)}{2}$$

$$\Rightarrow R_{ch}(\cos(at).f(t)) = \frac{F_C((p+ia),T)+F_C((p-ia),T)}{2i}$$

Theorem 3.8: If $R_{ch}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{ch}(\sin(at).f(t)) = \frac{-F_S((p-ia),T)+F_S((p+ia),T)}{2i}$$

Proof: Using Theorem 3.4, we have

$$R_{ch}(\sinh(at).f(t)) = \frac{-F_S((p-a),T)+F_S((p+a),T)}{2}$$

$$\Rightarrow R_{ch}(\sinh(iat).\,f(t)) = \frac{-F_S((p-ia),T)+F_S((p+ia),T)}{2}$$

$$\Rightarrow R_{ch}(\sin(at).f(t)) = \frac{-F_S((p-ia),T)+F_S((p+ia),T)}{2i}$$

Theorem 3.9: Suppose $f(t) = 0$, for $t < 0$. If $R_{sh}(f(t)) = F_S(p,T)$ and $R_{ch}(f(t)) = F_C(p, T)$, then

$$R_{ch}(f(t-a)) = \cosh(pa)\, F_C(p, (T-a)) + \sinh(pa)\, F_S(p, (T-a))$$

Proof: Let $R_{sh}(f(t)) = F_S(p, T)$ and $R_{ch}(f(t)) = F_C(p, T)$, Then

$$R_{ch}(f(t-a)) = \int_0^T f(t-a)\cosh(pt)\,dt$$

$$= \int_0^{T-a} f(x)\cosh(p(a+x))\,dx$$

$$= \int_0^{T-a} f(x)\,[\cosh(pa).\cosh(px) + \sinh(pa).\sin(px)]\,dx$$

$$= \cosh(pa) \int_0^{T-a} f(x)\cosh(px)dx + \sinh(pa) \int_0^{T-a} f(x)\sin(px)\,dx$$

$$= \cosh(pa)\, F_C(p, (T-a)) + \sinh(pa)\, F_S(p, (T-a))$$

4. EXAMPLES:

4.1 Find $R_{ch}(t \cosh(t))$

Solution: We know that

$$R_{ch}(t) = \frac{T\sinh(pT)}{p} - \frac{\cosh(pT)-1}{p^2}$$

and $$R_{ch}(\cosh(at).f(t)) = \frac{F_C((p+a),T)+F_C((p-a),T)}{2}$$

$$\Rightarrow R_{ch}(t\cosh(t)) = \frac{\dfrac{T\cosh((p+1)T)}{(p+1)} - \dfrac{\cosh((p+1)T)-1}{(p+1)^2} + \dfrac{T\cosh((p-1)T)}{(p-1)} - \dfrac{\cosh((p-1)T)-1}{(p-1)^2}}{2}$$

4.2 Find $R_{ch}(t \sinh(t))$

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

and $$R_{ch}(\sinh(at).f(t)) = \frac{-F_S((p-a),T)+F_S((p+a),T)}{2}$$

$$\Rightarrow R_{ch}(t\sinh(t)) = \frac{\dfrac{-T\cosh((p-1)T)}{(p-1)} + \dfrac{\sinh((p-1)T)}{(p-1)^2} + \dfrac{T\cosh((p+1)T)}{(p+1)} - \dfrac{\sinh((p+1)T)}{(p+1)^2}}{2}$$

4.3 Find $R_{ch}(te^t)$

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

and $$R_{ch}(e^{at}.f(t)) = \frac{F_C((p-a),T)+F_C((p+a),T)-F_S((p-a),T)+F_S((p+a),T)}{2}$$

$$\Rightarrow R_{ch}(te^t) =$$

$$\frac{T\cosh((p+1)T)}{(p+1)} - \frac{\sinh(p+1)T}{(p+1)^2} - \frac{T\cosh((p-1)T)}{(p-1)} + \frac{\sinh((p-1)T)}{(p-1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{\sinh((p-1)T)}{(p-1)^2} + \frac{T\cosh((p+1)T)}{(p+1)} - \frac{\sinh((p+1)T)}{(p+1)^2}$$

4.4 Find $R_{ch}(te^{-t})$

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

and $R_{ch}(e^{-at}.f(t)) = \dfrac{F_C((p-a),T)+F_C((p+a),T)+F_S((p-a),T)-F_S((p+a),T)}{2}$

$\Rightarrow R_{ch}(te^{-t})$

$$= \frac{\frac{T\cosh((p+1)T)}{(p+1)} - \frac{\cosh((p+1)T)-1}{(p+1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{\cosh((p-1)T)-1}{(p-1)^2} + \frac{T\cosh((p-1)T)}{(p-1)} - \frac{\sinh((p-1)T)}{(p-1)^2} - \frac{T\cosh((p+1)T)}{(p+1)} + \frac{T\sinh((p+1)T)}{(p+1)^2}}{2}$$

4.5 Find $R_{ch}(t\cos(t))$

Solution: We know that

$R_{ch}(t) = \dfrac{T\sinh(pT)}{p} - \dfrac{\cosh(pT)-1}{p^2}$ and $R_{ch}(\cos(at).f(t)) = \dfrac{F_C((p+ia),T)+F_C((P-ia),T)}{2}$

$$\Rightarrow R_{ch}(t\cos(t)) = \frac{\frac{T\cosh((p+ia)T)}{(p+i)} - \frac{\cosh((p+i)T)-1}{(p+i)^2} + \frac{T\cosh((p-i)T)}{(p-i)} - \frac{\cosh((p-i)T)-1}{(p-i)^2}}{}$$

4.6 Find $R_{ch}(t\sin(t))$

Solution: We know that

$$R_{sh}(t) = \frac{T\cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}$$

and $R_{ch}(\sin(at).f(t)) = \dfrac{-F_S((p-ia),T)+F_S((p+ia),T)}{2i}$

$$\Rightarrow R_{ch}(t\sin(t)) = \frac{-\frac{T\cosh((p-i)T)}{(p-i)} + \frac{\sinh((p-i)T)}{(p-i)} + \frac{T\cosh((p+i)T)}{(p+i)} - \frac{\sinh((p+i)T)}{(p+i)^2}}{2i}$$

4.7 Find $R_{ch}(\sin(5t))$

Solution: We know that

$R_{sh}(\sin(t)) = \left(\dfrac{-1}{p^2+1}\right)\sin(pT).\cos(T) + \left(\dfrac{p}{p^2+1}\right)\cosh(pT)\sin(T)$ and $R_{sh}(f(at)) = \dfrac{F_S(\frac{p}{a},aT)}{a}$

$$\Rightarrow R_{sh}(\sin(5t)) = \left(\frac{-5}{p^2+25}\right)\sinh(pT)\cos(5T) + \left(\frac{p}{p^2+25}\right)\cosh(pT)\sin(5T)$$

4.8 Find $R_{ch}(\sin(5t))$

Solution: We know that

$$R_{ch}(\sin(t)) = \left(\frac{1}{p^2+1}\right)[1-\cosh(pT).\cos(T)] + \left(\frac{p}{p^2+1}\right)\sinh(pT)\sin(T)$$

and $R_{ch}(f(at)) = \dfrac{F_C(\frac{p}{a}, aT)}{a}$

$$\Rightarrow R_{ch}(\sin(5t)) = \left(\frac{5}{p^2+25}\right)[1-\cosh(pT)\cos(5T)] + \left(\frac{p}{p^2+25}\right)\sinh(pT)\sin(5T)$$

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