STUDY OF RESISTANCE IN STEADY GRADUALLY VARIED FLOW PROFILES

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ABSTRACT

The paper describes the numerical solution of gradually varied flow with the logarithmic resistance explicit formula of Colebrook–White equation of pipe flow in the open channel replacing pipe diameter by the equivalent diameter of open channel in order to study the effect of transition turbulent effect on the flow profiles. The model solution obtained taking Manning's equation as resistance equation is compared graphically with the available experimental data. The plot in four different profiles has shown a close agreement between experimental data and model solution. This has justified the use of channel resistance equation which takes care of flow in different state of flow.

Keywords: *Manning's n, Chezy's C, Friction factor f, Equivalent diameter D_e, Reynold's no. R_e, Friction slope S_f ,*
Bed slope S_b.

INTRODUCTION

Occurrence of steady gradually varied flow in the practical field is quite common. A vast work on the solution of the nonlinear differential equation of this flow has also been found in literature. But the resistance equation used frequently by the previous researchers is Manning's formula taking flow to be turbulent. The flow condition in gradually varied flow may vary from laminar to transition and to turbulent states of flow. Use of Manning's equation is justified when flow is fully turbulent. Effect of resistance to the gradually varied flow profiles is not truly predicted by taking the constant value of Manning's n when the flow conditions enter in laminar and transition zones with different roughness patterns. Therefore, effect of resistance in steady gradually varied flow profiles is presented using an equation of direct explicit computation of implicit Colebrook-White (C-W) resistance equation developed by Barr in the numerical solution of the governing equation of gradually varied flow. Solution obtained is presented for M_1, M_2, H_2 and A_2 profiles to show the effect of resistance for different types of resistance equation

GOVERNING EQUATION OF STEADY GRADUALLY VARIED FLOW

The governing differential equation of gradually varied flow is written with usual notation of open channel flow as:

$$\frac{dy}{dx} = \frac{S_b - S_f}{1 - \alpha \frac{Q^2 T}{g A^3}} \tag{1}$$

Assuming the energy coefficient α to be unity, the equation is written as:

$$\frac{dy}{dx} = \frac{S_b - S_f}{1 - \frac{Q^2 T}{g A^3}} \tag{2}$$

Solution or computation of the above differential dynamic or governing equation gives types of profiles depending on the bed slopes and zones in which occur, length and shape of profiles. But this equation cannot be integrated directly. It is nonlinear as the as the dependent variable y is a function of both x and y. Hence, equation (2) may be written as:

$$\frac{dy}{dx} = \frac{S_b - S_f}{1 - \frac{Q^2 T}{g A^3}} = f(x, y) \tag{3}$$

The friction slope: $$S_f = \frac{fV^2}{2gD_e} \tag{4}$$

where f is the friction factor and D_e is equivalent diameter of open channel which is equal to four times of area divided by wetted perimeter i.e.

$$D_e = \frac{4A}{P} \tag{5}$$

The evaluation of friction factor f is done in this study by a suitable logarithmic generalised C-W equation[1,] developed by Barr[2].

PREVIOUS WORKS ON SOLUTION

Belanger's[3] name is mentioned to be the first who used method of successive approximation. Dupuit[4] attempted for the first time to integrate the equation. Bresse's[5] solution was only for wide rectangular channel. Bakhmeteff[6] solved with varied flow functions. Chow[7,8] methods is the outcome of many previous existing methods with modification. Computation of profiles by different numerical methods is due Pickward[9], Vallentine[10], Cheng and Wang[11], Eichert[12], Prasad[13], Roa and Sridharan[14], Minton and Sobey[15], Kumar[16], Choudhury and Schulte[17] and Das and Saikia[18]. Das and Saikia conducted experimental woks and compared their solution with Manning's with their experimental data. In the solution of above investigators, the resistance equation used was Manning's or Chezy's formula which does not represent the resistance in the laminar and transition zone flow specially in backwater curves like M_1, S_1 profiles where velocity of flow decreases owing to the barrier along the flow direction.

RESISTANCE EQUATIONS USED

Prandtl[19] deduced the following logarithmic resistance equation in terms of Darcy's friction factor f and Reynolds number R_e for smooth pipes from the experimental data of Nikurade.

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{\sqrt{f}}{2.51}R_e\right) \tag{6}$$

Nikuradse[20] had shown that if the walls of the pipe were rough and the Reynolds number R_e is sufficiently high, f became independent of R_e and dependent on relative roughness only. He evaluated the following formula for f in terms of ratio of diameter D of the pipe to the diameter of sand grains k that he used to roughen the inside of the pipe.

$$\frac{1}{\sqrt{f}} = 2\log_{10} 3.71\left(\frac{D}{k}\right) \tag{7}$$

Colebrook and White[1] combined above two smooth and rough friction factor equations into a single equation as:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.71D} + \frac{2.51}{R_e\sqrt{f}}\right) \tag{8}$$

Equation (8) is called Colebrook-White (C-W) equation, which can be used for both smooth and rough commercial pipes. In equation (8), if R_e is sufficiently high, the second term within the bracket is negligible and the equation becomes equal to equation (7). Similarly if the inside wall is smooth, the first term within the bracket is zero and the equation becomes equation to equation (6). Besides, this equation takes care of flow condition from transition to turbulent zones. If the pipe diameter is replaced by equivalent diameter D_e of open channel which four times the hydraulic radius R of the channel i.e. D=4R, equation (7) for rough turbulent open channel flow becomes:

$$\frac{1}{\sqrt{f}} = 2\log_{10} 3.71\left(\frac{4R}{k}\right) \tag{8}$$

Similarly C-W white equation for open channel flow becomes with Reynolds number $R_e = \frac{4VR}{\upsilon}$:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{14.84R} + \frac{2.51}{\frac{4VR}{\upsilon}\sqrt{f}}\right) \tag{9}$$

Direct application of equation (9) is implicit and hence computation of friction factor requires a rigorous method of iteration. An explicit approximation to this formula is given by Barr[2], which is written as:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{14.84R} + \frac{5.1286}{R_e^{\ 0.89}}\right) \tag{10}$$

Equation (10) gives a very good assessment in the simulation of unsteady surge tank water level fluctuations by Saikia et al[21], Saikia and Sarma[22] in their model "UNSTD_FRIC-BARR_WH"

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using MoC and Barr and Das[23] in unsteady dam-break profiles. Simulation of profiles in all the situations agrees quite well with experimental data. Therefore, in the proposed study, resistance equation (10) is used to predict the simulation of steady gradually flow profiles in transition-turbulent states of flow.

NUMERICAL METHOD USED FOR SOLUTION

Different numerical solutions are available for solution of this type of differential equations. Das[24] has made a detailed analysis of six different numerical method of solution. He has concluded that trapezoidal integration method gives a better solution. This method has also been used by previous investigators for computation of the profiles by Manning's equation. Therefore, this method of solution has been adopted in this study. Details of this method have been referred to Das[24].

SIMULATION OF RESULTS

Simulation of numerical solution is done by comparing the author's experimental data and the theoretical solution. The author conducted experiment in a laboratory channel of 6 m long and 0.10 m wide. The channel is equipped with a venturimeter to measure the discharge and has the facility to adjust the bed slope with a screw-jack. The value of Nikuradse's sand roughness size k of the channel has been calibrated to use in approximate C-W equation. Coloured water was recirculated with a pipe and pumping system. Control section in the form of weir, free overfall and sluice gate were provided in the flume to generate data for the profiles. Authors' solution with approximate C-W resistance formula has shown better agreement with experimental data than the solution of turbulent Manning's resistance formula. The profiles with Manning's have given lower depth. This happens due to lower resistance to flow. Manning's equation cannot predict the correct assessment when Reynolds number of flow deceases. On the other hand C-W equation takes care of resistance from transition to turbulent state of flow.

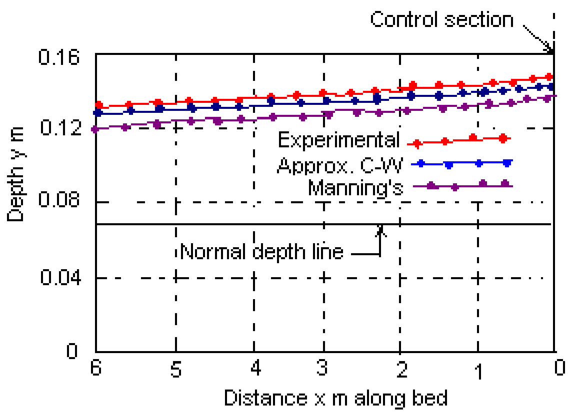

Figure 1 Comparison of M_1 profiles

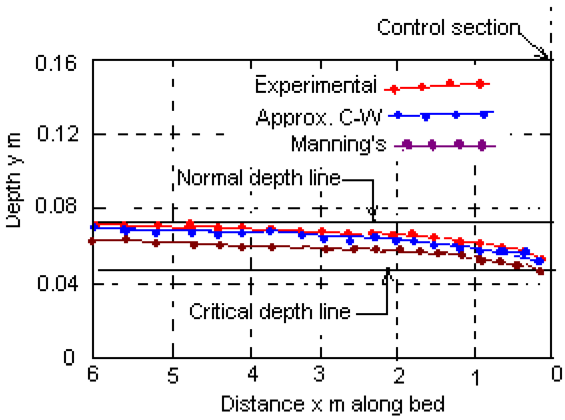

Figure 2 Comparison of M_2 profiles

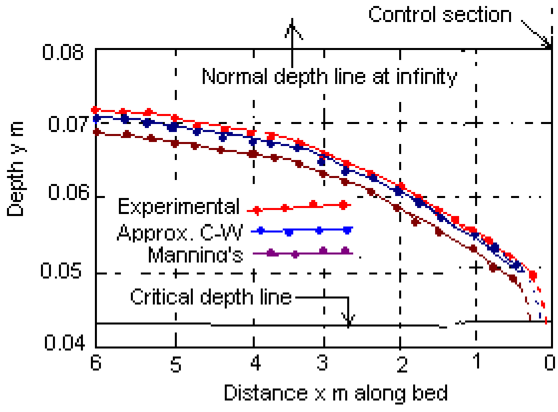

Figure 3 Comparison of H_2 profiles

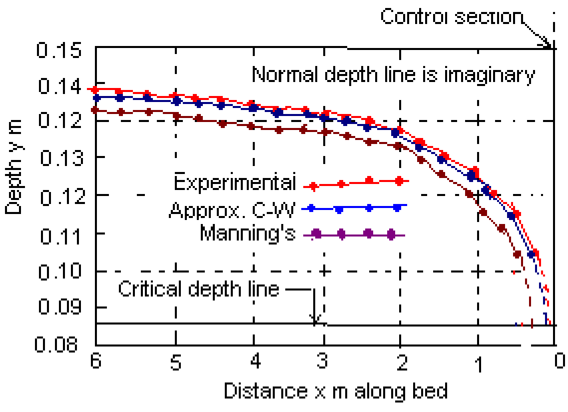

Figure 4 Comparison of A_2 profiles

CONCLUSION

The close agreement of authors' solution from figure 1 to figure 4 justifies the use of approximate C-W resistance equation in open channel flow situation. Comparison further reveals that although Manning's equation is widely used, it underestimate the resistance to flow. The authors' use of approximate C-W equation offers better resistance.

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